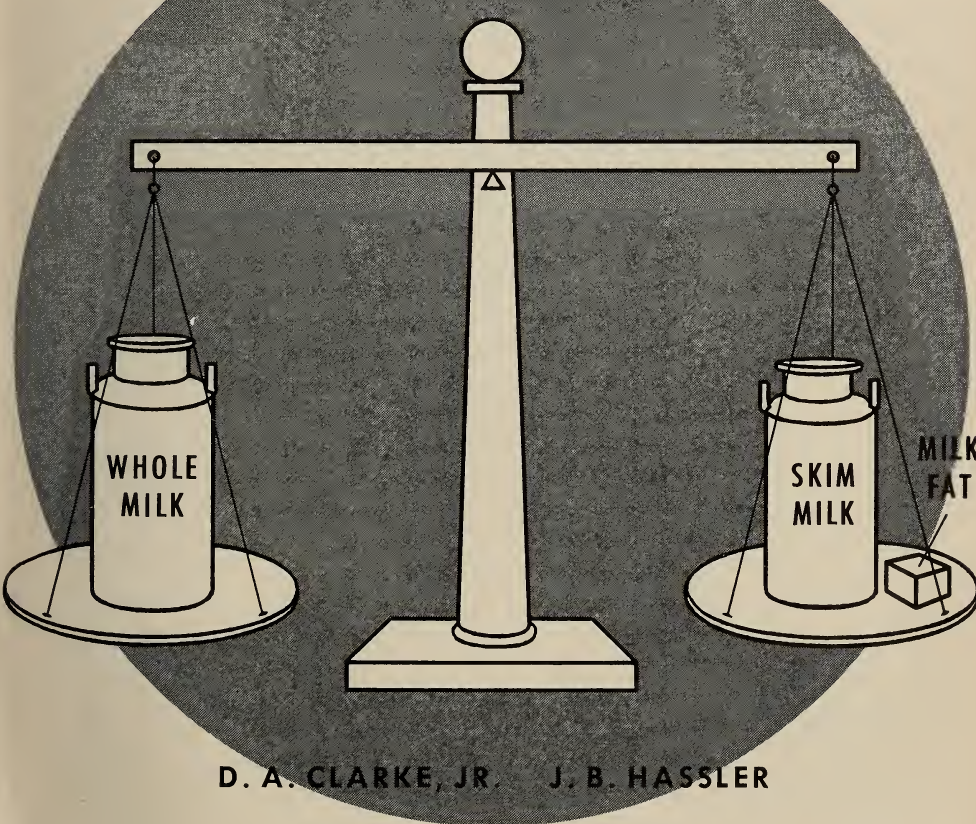
Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

PRICING FAT and SKIM COMPONENTS OF MILK



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Dairy scientists have known for some time that a general relationship exists between the amount of fat and nonfat solids in milk. Until recently, in evaluating whole milk, emphasis has been placed largely on the fat content. Since the early years of World War II, however, the nonfat constituent has become increasingly important. This shift in emphasis has complicated the problem of establishing equitable producer prices for whole milk of varying fat and nonfat solids content.

A pricing system that would treat all producers equitably, regardless of fat content of the raw milk sold to processors, is suggested in this bulletin. Such a system must "price" both fat and skim components as nearly as possible on their real values. A series of equations is presented herein which may be used as a guide to establishing these values. Under a pricing plan of this type, a producer would not be penalized for milk of specified fat content, nor would he feel forced to augment purebred stock with other breeds in order to raise the average fat content output of his herd.

In addition, inequities exist under classified pricing of market milk where accounting for utilization is based solely on the fat constituent. This bulletin further suggests methods of establishing separate prices for fat and skim which, if adopted, would serve to correct these inequities.

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PRICING FAT and SKIM[1] COMPONENTS of MILK

D. A. CLARKE, Jr. • J. B. HASSLER

VARIATIONS IN MILK COMPOSITION

THE PROBLEMS of establishing producer prices for milk are made more complex by variations in its physical and chemical composition. Milk sold by producers commonly varies in content from about 3 per cent milk fat to nearly 6 per cent. Furthermore, the nonfat solids content of "normal" milk may range from a little under 8½ to almost 10 per cent. Since the values of all components enter into the value—or price—of whole milk, variations in constituents must be considered in determining a pricing schedule if it is to be fair and equitable to all concerned.

It has long been recognized that a given amount of milk has a greater value when it contains a large rather than small amount of milk fat. Because of this, the pricing systems in effect in most markets in the United States include a "butterfat differential."[2] As most milk is sold by weight, this differential provides for a higher price per hundredweight for milk of high fat content than for the same amount of milk with a lower fat content.

More recently—particularly since the early years of World War II—increased emphasis has been placed on the relative importance of the nonfat constituent of milk. This is true in both the manufacturing and the market milk industries. In the former, it is reflected in the increased importance of whole milk and

[1] This bulletin presents the results of work which was undertaken by the University of California in response to Senate Resolution No. 74 of the 1947 California Legislative Session. The determination of the physical relationship between fat and nonfat solids was carried on by the Department of Dairy Industry under the supervision of Professor E. L. Jack. The pricing problems were undertaken by the Department of Agricultural Economics. Statements concerning the bearing of this problem on pricing policies have previously been made by members of the latter department at public milk hearings. The present authors wish to acknowledge the contributions of Professors R. G. Bressler and J. M. Tinley.

[2] The butterfat differential is usually defined as the amount to be added or subtracted from the base price per hundredweight for each 1/10 of 1 per cent above or below the basic fat content—which in California is 3.8 per cent. In markets where a "direct ratio" price system—payment is made solely on the amount of fat content—is used (such as in California for Grade A milk, prior to 1948), the butterfat differential would be 1/10 the price of a pound of milk fat.

skim milk products relative to that of butter. On the market—or fluid milk—side, the same type of change can be noted. The average fat content of fluid milk has been decreased, sales of "heavy" cream have given way to lower fat items—especially "half-and-half"[3]—and ice cream sales have not kept up with increases in the sales of ice milk.

Pricing milk on the basis of fat test has been made possible largely through the use of the Babcock Test. Developed in 1890, this test provides a reasonably accurate, simple, and inexpensive means by which the percentage of milk fat in a given sample of milk can be determined. On the other hand, there is at present no practically adaptable test by which the nonfat solids content of each individual shipment can be measured.[4] Accurate and reliable methods of determination of nonfat solids do exist, but they are expensive and time consuming.

Dairy chemists have long been aware of a general relationship between the amount of fat and nonfat solids in milk. Milk with a relatively high fat content also contains relatively large amounts of nonfat solids. Furthermore, on the basis of the results of a large number of sample cases, a reasonably reliable average relation can be determined. This average relationship can then be used to estimate the nonfat solids in milk of any specified fat content.

Perhaps the best known study of this fat-nonfat solids relationship has been Jacobson's analysis of 150,000 samples of milk produced in New England.[5] This indicated that a 1 per cent difference in fat content was, on the average, associated with a difference in the same direction of $\frac{4}{10}$ of 1 per cent in the amount of nonfat solids. Milk with a fat test of 4 per cent contained, on the average, 8.67 per cent nonfat solids, while milk with 5 per cent fat would have 9.07 per cent of these associated serum solids.

Due to the importance of the problem of pricing the nonfat milk components, the Dairy Industry Department of the College of Agriculture, University of California, has recently completed a study of the composition of milk produced under California conditions.[6] In all, nearly 21,000 samples of milk were analyzed during a two-year period. These samples were taken every two weeks from shipments received at processing plants from representative shippers of both manufacturing and market grade milk. For the purpose of obtaining the samples and analyzing the results, the state was divided into 10 separate areas. These areas, in turn, were selected to provide reasonable uniformity in types of feeding and management practices and in predominant breeds of cattle.

On the basis of the samples so collected, the authors determined—by mathemati-

[3] The term used by the industry for a product which is half table cream and half milk, with a resulting milk fat content of approximately 12 per cent.

[4] The lactometer—which has often been used to measure the nonfat solids content of milk—has not been considered sufficiently accurate for this purpose. This technique is based on differences in the specific gravity of milk with high and low solids content. These nonfat solids, however, are not homogeneous, but include all of the elements of milk other than fat and water—such as casein, albumen, sugar, and ash. Changes in the proportions of these nonfat solids elements influence the specific gravity, and so influence the accuracy of the tests. This method has not been officially accepted by the Association of Official Agricultural Chemists. At the time of writing, it has been announced that the University of Maryland and the United States Department of Agriculture are coöperating on a project to develop more accurate lactometer tests.

[5] This is the so-called "Jacobson relation" which, in mathematical terms, can be expressed as:

$SNF = 7.07 + 0.40F$, where SNF is the percentage of nonfat solids and F, the percentage of fat content. (Reported in *Journal of Dairy Science*, **XIX**, 1936, 171–76.)

[6] For a more complete report of the results of this study, see "Relationship of Solids-not-fat to Fat in California Milk," by E. L. Jack, E. B. Roessler, F. H. Abbott, and A. W. Irwin. Calif. Agr. Exp. Sta. Bul. 726, 1951.

cal means—a linear, weighted, composite relationship that differs slightly from Jacobson's results. In equation form, this is:

$$NFS = 7.07 + 0.444F$$ [7]

where NFS represents the amount of nonfat solids, and F the amount of milk fat present in a hundredweight of milk. This same relationship is shown graphically in Figure 1. From this relationship it can be seen that milk of 3.8 per cent fat content contains, on the average, 8.75 pounds of nonfat solids per hundredweight. For each change of $1/10$ per cent in fat content, the nonfat solids change (in the same direction) by 0.0444 per cent. Therefore, on the average, milk with 3.9 per cent fat will contain 8.80 per cent nonfat solids, while that with 3.7 per cent fat will contain only 8.71 per cent nonfat solids. This relationship is considered applicable through all ranges of fat test analyzed, that is, from 3 per cent to 6 per cent.

Figure 2 presents this same relationship, and provides a comparison with other procedures commonly used in milk payment plans. These include the previously mentioned "Jacobson relation" and a representation of a "direct ratio" between fat and nonfat solids. The latter case depicts a situation in which the nonfat solids increase in a direct and constant proportion with increases in fat, and where the ratio of fat to nonfat solids is arbitrarily determined at the level indicated by the California relation for milk of 3.8 per cent fat content. For milk of this test, there are 2.3 pounds of nonfat solids per pound of milk fat.

Also shown in Figure 2 are points which represent averages of fat and nonfat solids content for groups of samples of milk produced by herds classified according to predominant breeds.[8] As can be seen from the "scatter" of these points, the use of the linear function represented by the line labeled "California relation" is at best an approximation for determining nonfat solids on the basis of fat content.[9] It can be noted that Holstein milk of 4.2 per cent fat content (a high test for the breed) averaged slightly less than 8.8 per cent nonfat solids, while Jersey milk of the same fat test (a relatively low test for this breed) averaged more than 9.2 per cent nonfat solids. Furthermore, minor inconsistencies exist between these *observed* averages within breeds due to sampling variability. For example, Jersey milk samples of 4.8, 4.9, and 5.0 per cent average fat content all contained the same amount of nonfat solids, while milk from the same breed of cows, that averaged 4.7 per cent fat, contained approximately one-half per cent less nonfat solids.

In spite of the variation apparent in Figure 2, two points can be made to justify the adoption of the California relationship for pricing purposes under current conditions. These are:

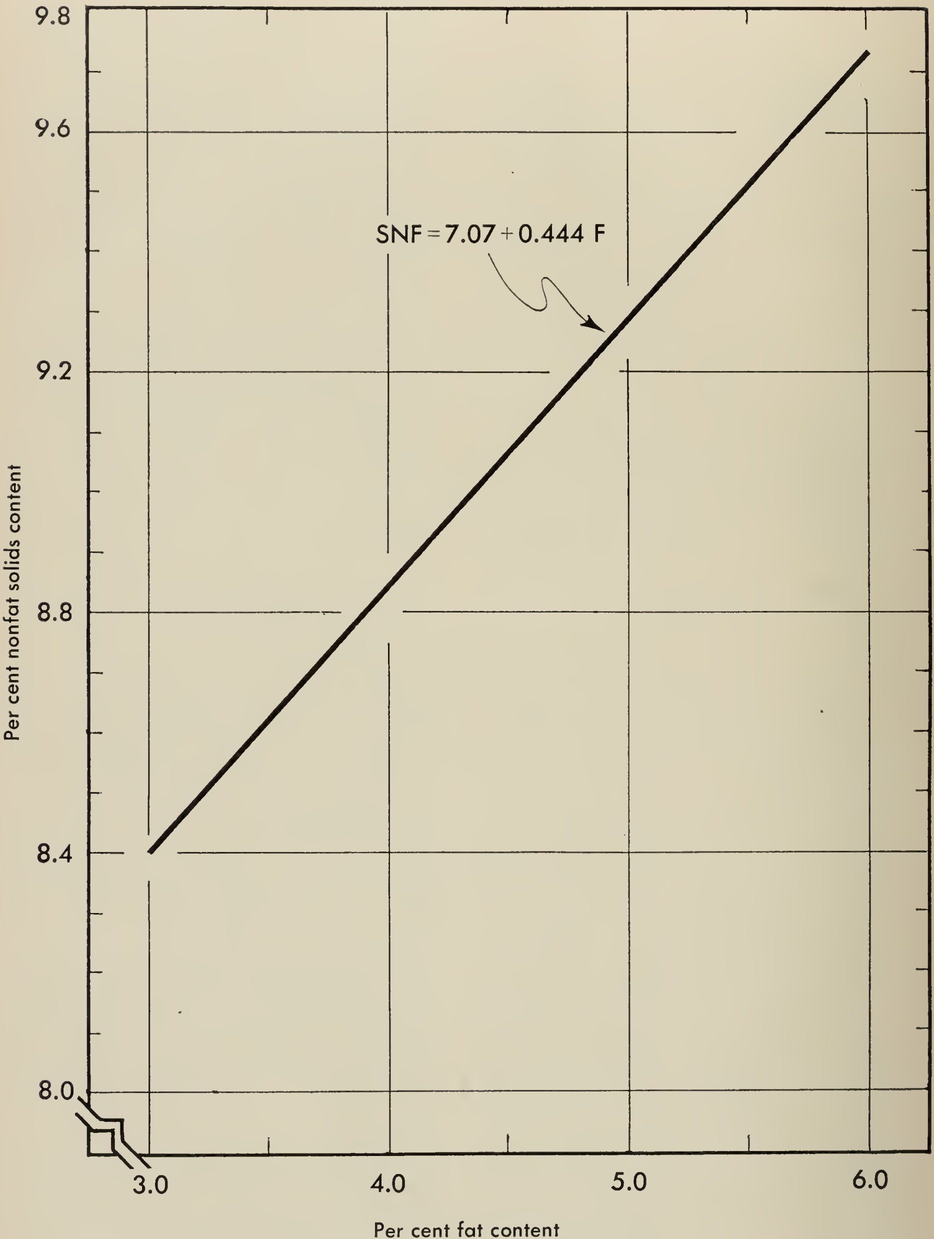
1. The greatest departure of the actual averages from the estimating relation occurs at the high and low limits of fat content for the particular breed under consideration. For example, the error

[7] As an indication of the reliability of this relationship, the authors have further determined that, on the basis of chance alone, they would expect an error between estimated (based on this mathematical equation) and actual (based on individual laboratory determination) nonfat solids content that would be greater than 0.71 per cent only once in 20 tests. If, for example, a series of tests were to be made of groups of 20 samples, it would be expected that, on the average, only one sample in each 20 would result in a difference between estimated and actual nonfat solids of as much as 0.71 per cent in either direction. Furthermore, if weekly composite samples were used, only one in 20 tests would have an error of greater than 0.29 per cent.

[8] These group averages were determined by Professor Jack and his associates, and made available to the present writers for this purpose. These data do not appear in Bulletin 726, from which all other information relative to the composition of milk produced under California conditions has been drawn.

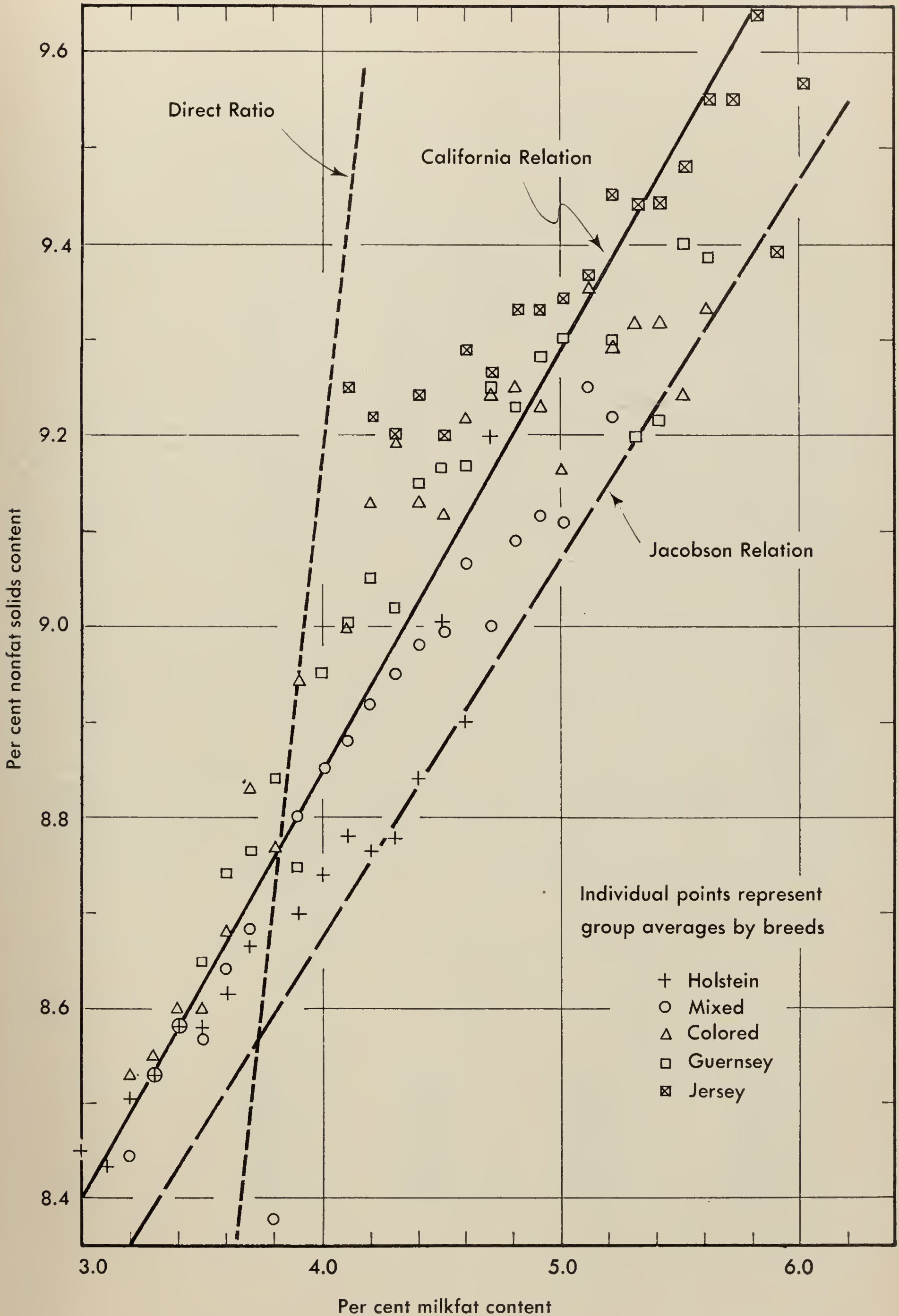
[9] The statistical measure of the reliability of this relation and estimates of the magnitude of error resulting from the use of this relation as an estimating equation have been presented in footnote 7.

FIGURE 1. RELATIONSHIP OF SOLIDS-NOT-FAT TO FAT IN CALIFORNIA MILK*



* Source: California Agricultural Experiment Station Bulletin 726.

FIGURE 2. RELATIONSHIP BETWEEN THE FAT AND NONFAT SOLIDS CONTENT OF CALIFORNIA MILK



of the greatest magnitude occurs for Jersey milk of approximately 4.0 per cent fat content. By far the bulk of Jersey milk produced has a much higher fat content—near the average test for the breed—which is more than 5 per cent. For any particular breed, the linear relation provides a very close approximation to the *normal* average fat test for that breed. Futhermore, over a period of time, the average fat test for a herd of a given breed will shift around the normal average fat test for that breed, so that compensating errors will tend to balance.

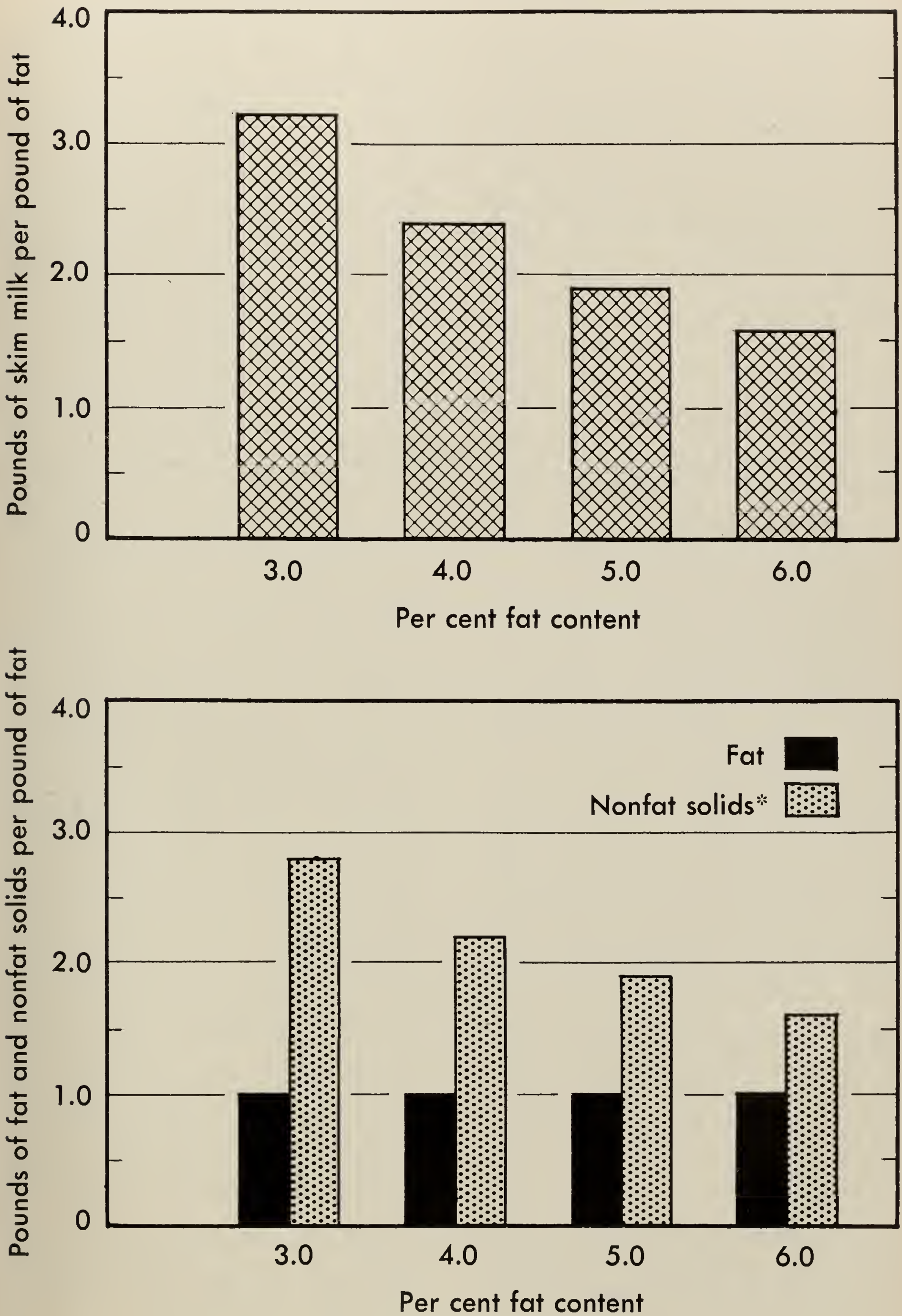
2. The California relation was mathematically determined to be the "best fitting" straight line describing the average relationship between fat and nonfat solids in milk produced under California conditions.[10] As can be seen from Figure 2, it provides a better representation than does the Jacobson relation which consistently results in a lower estimate of nonfat solids for any level of fat content. In addition, the California relation is a substantially better basis for pricing than is the direct ratio procedure which overpays for milk of higher than the average fat content while underpaying for the below-average tests of milk in relation to the available evidence concerning nonfat solids content.

[10] By the method of "least squared residuals."

In view of the above and of the aforementioned difficulties of direct determination, it appears that the California relation provides the best available information for use in estimating nonfat solids content of milk, and it will therefore be used in the development of pricing plans discussed herein. It should be emphasized, however, that when new methods of determination become available, either in the form of new and improved estimating procedures or in the form of techniques for direct measurement, these more accurate devices can, and should, be substituted in the pricing process.

Figure 3 shows the different volumes of milk and the amounts of fat and nonfat solids which are associated with various types of milk per pound of milk fat. For every pound of fat in 3 per cent milk, there are 32.3 pounds of skim milk which contain an average of 2.8 pounds of nonfat solids. Correspondingly, a pound of fat in 4 per cent milk is associated with 24 pounds of skim milk and some 2.2 pounds of nonfat solids; 5 per cent milk has 19 pounds of skim milk and an average of 1.9 pounds of nonfat solids; while a pound of fat in 6 per cent milk has only 15.7 pounds of skim milk and about 1.6 pounds of nonfat solids per pound of fat.

FIGURE 3. CONSTITUENTS PER POUND OF MILKFAT OF MILK OF VARIOUS LEVELS OF FAT CONTENT



* Estimate of nonfat solids based on California relationship: NFS = 7.07 + 0.444 F

PRICING BASED ON DIFFERENTIAL VALUES

The major premise upon which this report is based is that a pricing system or schedule should be designed to reflect as nearly as possible the real values of milk of varying fat and nonfat solids content. To accomplish this, the pricing plan should "price" both the fat and skim milk components realistically—according to values of the constituents as set by the market. Under such an ideal system, where standardization is legal, it would make no difference, in raw product cost to a processor, whether milk of high or low fat content were received. If the incoming milk from producers contained too much fat relative to the skim components, the surplus fat could be sold—or conversely, additional skim milk could be purchased—at prices consistent with those paid for the different types of whole milk.[11] Furthermore, there would be no basis for discriminating against producers of milk with high or low fat content. Nor would there be any economic pressures for breeders of purebred stock to add cows of different breeds and so influence the average fat content of the herd.

Manufacturing Grade Milk Prices.—The solids content of milk has a direct influence on the yields of manufactured dairy products and so on the problem of pricing milk used for this purpose. Where differences in product yields occur, the net return from the sale of products by the manufacturer is affected. For this reason, the processing plant can afford to pay producers a higher price for milk which yields relatively large amounts of products per hundredweight than for that which has low product yields.

Table 1 shows how the yields of products from a butter-powder operation and from a condensery operation vary with different levels of solids content. For this purpose, the amounts of nonfat solids have been calculated according to the California relationship described in previous sections. The physical yields of the different manufactured dairy products have been calculated on the basis of legal standards set for the fat and nonfat solids content of these final products. Adjustments have been made for such plant losses of solids as typically occur in reasonably efficient processing plants. The different types of by-products derived from the condensery operation—butter and powder—result from the standardization of milk going into evaporated milk. Whole milk of 3.9 per cent fat normally contains fat and nonfat solids in about the ratio legally required for this product. Milk of lower test is standardized to the desired ratio by reducing the proportion of nonfat solids, that is, by separating out skim milk which, in this example, is then made into dried, nonfat solids milk powder. Milk of higher test contains an excess of fat relative to nonfat solids, and so in this case cream is removed and made into butter.[12]

Because the solids content of milk enters so directly into the determination of product yields, the value of these differences in yields can be used to determine pricing schedules. Given the yields and f.o.b. plant product prices, adjustment allowances for processing costs could be made, and the result would provide the *net value*, or "paying ability" of milk of different fat tests, to the processing plant. These net values rise

[11] As will be discussed later, with any given price schedule for whole milk there is only one set of prices that will accomplish this consistency. They establish specific relationships among the price of whole milk, the separate prices of fat and skim milk, and the butterfat differential.

[12] It is also true that this standardization could take place through purchases of skim milk or cream from outside plants. With a competitive market for these components—that is, consistent market prices—the result would be equivalent to standardizing within the plant as indicated above.

Table 1. Calculated Yields of Manufactured Dairy Products per Hundredweight of Milk of Varying Solids Content—Butter Powder and Condensery Operations*

Fat content	Nonfat solids content	Butter powder		Condensery		
		Butter	Powder	Evaporated milk	Powder	Butter
		pounds per hundredweight of whole milk				
3.0	8.40	3.57	8.49	36.98	1.59	....
3.5	8.62	4.18	8.71	43.30	0.66	....
4.0	8.85	4.80	8.93	47.94	....	0.18
4.5	9.07	5.41	9.15	48.22	....	0.80
5.0	9.29	6.03	9.38	48.50	....	1.42
5.5	9.51	6.64	9.60	48.79	....	2.04
6.0	9.73	7.26	9.82	49.07	....	2.66

* Hypothetical examples of plant operations in which it is assumed that the use of milk in the former operation is solely for butter and nonfat dry milk solids and that in the latter operation, the principal product is evaporated milk with a by-product operation of butter or powder utilizing the residual resulting from standardization.

according to increased product yields associated with the higher levels of fat—and so nonfat solids—content. The amount of the value increase depends, of course, on the relative net prices of the specific products. Since there is no direct ratio between increases in product yield and increases in fat test (except in the butter part of the butter-powder operation), these net values will reflect differences which are not in direct proportion to differences in fat content. This means, essentially, that a hundredweight of 6 per cent milk is not so valuable as 200 pounds of 3 per cent milk. Yet, with a direct ratio-to-fat pricing system—such as used by many manufacturing milk plants—the same amount of money is paid for these different quantities of low- and high-test milk. Plants which pay producers a fixed amount per pound of butterfat, in other words, are penalizing low fat producers and overpaying high fat producers relative to the "net value" of the different types of milk received.

Milk used for manufacturing purposes can be priced according to the net value of the products obtained, as indicated in the above section. Under such a pricing system, differences in prices received by producers will directly reflect the differences in the values of the products manufactured from these milks.[13] In this sense, producers of milk of differing composition will be treated equitably with respect to the prices they receive. Unless these net values of milk of different fat and nonfat solids content are respectively equal in all economically alternative uses, however, there is no assurance that a *single* uniform payment schedule for a given area will be appropriate.[14]

[13] A similar argument relating to the determination of butterfat differentials is presented in the University of Wisconsin Research Bulletin 143, "Paying Producers for Fat and Solids-Not-Fat in Milk," by Rudolph K. Froker and Clifford M. Hardin, 1942.

[14] With reference to the "consistency" of prices of the various manufactured dairy products, it is of interest to note that a study of interproduct price relationships has recently been completed by the junior author. This study is currently in the process of publication under the title, "Pricing Efficiency in the Manufactured Dairy Products Industry." The conclusions indicate that, historically, the prices of

There are, of course, differential prices for milk at different levels of the marketing system. For example, the combined value of the fat and skim elements in cream and skim milk per hundredweight of whole milk is greater—by the cost of receiving and separating the milk—than the value, at the delivery platform, of the whole milk to be separated. Also, the value of milk at the receiving plant is higher—by the necessary transport cost—than the value of the same milk at the farm level. Because it is sometimes necessary or desirable, for payment purposes, to announce prices at different levels, that is, f.o.b. farm and f.o.b. plant, some adjustment must be made to convert these prices from one level to the other. This creates a problem in manufactured milk pricing—where milk values are considered related to the fat and nonfat solids content—of a different nature from that encountered in the fluid milk pricing field where the value of the nonfat elements may be considered as a function of the volume of skim milk.

The material presented here on manufacturing grade milk values develops "net values" for the separate components by assigning all costs that are specifically identifiable with the fat or with the nonfat solids directly to the cost influencing element. For example, costs of churning butter are assigned to the fat component, while powder drying costs are directly charged to the nonfat solids constituent. Some of the relevant costs involved are joint costs and are not uniquely identified with either one or the other component. Such costs are those associated with handling the whole milk—for example, receiving and separating costs, and costs of transport from farm to plant. Any attempt to deduct costs of this type from separate fat and nonfat solids prices determined at plant level will involve an arbitrary *and essentially unequitable* allocation of these joint costs. The major difficulty in attempting to make an arbitrary allocation of these costs, wherein they may then be directly charged to either the fat or the nonfat solids, is that costs of this type—such as transportation, receiving, and separating—are independent of the amount of fat or nonfat solids involved. The costs are a function of the volume or weight of milk handled and, since the fat and nonfat elements vary for any given volume of milk, the cost per unit of fat and nonfat solids varies among milk types having different tests. This means that the cost of transporting, receiving, and separating, per unit of fat and nonfat solids, is lower for high-test than for low-test milk. Any attempt to assign a constant unit cost to the specific item, therefore, will of necessity underprice the fat and nonfat solids in high-test milk and overvalue these solids in low-test milk *at the farm level* relative to the actual costs involved for a particular type. A system of direct allocation of farm-to-plant costs to the individual components would necessarily involve a direct butterfat differential that is lower at the farm level than at the plant.

A logical alternative to a procedure of quoting separate prices for fat and skim solids f.o.b. farm level, therefore, might involve the use of separate prices f.o.b. plant level wherein these separate prices are ultimately combined into a hundredweight basis, with deductions of these joint costs made according to the weight or to some other cost-affecting variable. The major impact of this procedure would be that the resulting on-farm prices would involve the same calculated butterfat differential as milk at the plant level. This does not mean, of course,

butter, cheese, and whole milk powder have been reasonably consistent. Evaporated milk price quotations—particularly for the so-called "advertised brands"—appear to be high relative to those of the other major manufactured dairy products. Under such circumstances, the "net value" of milk used for evaporated milk operations would be higher than that for butter and cheese.

either that producers receive the same prices for the fat and nonfat components at the farm as would exist at the plant, or that these joint costs do not ultimately become assigned to the value components of the milk. It merely means that the producer bears these charges in the manner in which they accrue—according to the volume or weight of whole milk transported and handled. Under this system, the producer will tend to receive, at the plant, the net value of the milk he produces less the necessary charges involved in moving his milk from farm to plant.

SUGGESTED FORMULAS FOR USE IN COMPUTING PRICES AND BUTTERFAT DIFFERENTIALS FOR MANUFACTURING GRADE MILK[15]

The above sections have put forth the logical argument that an equitable pricing system is one which reflects the net values of milk of different constituency and quality standards. To accomplish this, it is necessary that the following information be available: (1) the physical quantities of the relevant milk product or products obtainable from milks of varying constituency, and (2) the net value (price less processing and other appropriate costs, such as transportation and marketing) of the milk products so obtained.

Relationships have been developed from which it is possible to determine the product yields for several alternative types of dairy operations. Where manufactured dairy products—butter, nonfat dried milk solids, and evaporated milk—are concerned, federal standards for fat and nonfat solids content have been used, and allowances have been made for such losses as normally occur in reasonably efficient operating plants.[16] In a similar manner, the yields of cream and skim milk have been computed, based on the fat test of the cream, with allowances made for separating and other plant losses.

Butter-Powder Operations.—Milk entering a processing plant to be converted into butter and dried nonfat solids is first separated into cream and skim milk. Normally, there is about a $\frac{1}{10}$ of 1 per cent loss of fat in this separating operation, that is, this amount of fat remains with the skim milk and does not enter into the butter operation. The cream entering into the churn is, typically, about 40 per cent milk fat, and therefore contains some nonfat solids. These solids can be reclaimed from the residual buttermilk and aggregated with the solids obtained directly from the skim milk, and in these calculations it has been assumed that this procedure is followed.

Considering standard allowances for moisture, and with typical product losses during the processing operation, the quantities of butter that can be obtained from a hundredweight of milk may be represented:

$$Q_b = 1.23F - .123$$

which means that each pound of fat in a hundredweight of milk will produce 1.23 pounds of butter, but that the over-all loss of fat in the separating operation (0.1 per cent) amounts to the equivalent of .123 pound of butter. The dried nonfat solids which can be obtained from

[15] The theoretical basis for the procedure in this section is essentially the same as that presented for establishing butterfat differentials for butterpowder and cheese factories by Froker and Hardin (see footnote 13). The major differences involved are in terms of the development of yield formulations and the extensions of the net value concept to include evaporated milk operations.

[16] For a complete discussion of the development of these yield relationships, including the federal standards and the operating loss allowances, see Hassler, James B., "Pricing Efficiency in the Manufactured Dairy Products Industry," Appendix A (in press).

this joint butter-powder operation may be determined by:

$$Q_{nfs} = 7.17 + .441F$$

where .441 represents the added pounds of dried nonfat solids that can be produced from the additional nonfat solids in whole milk which are associated with a variation of 1 per cent in fat content, as determined from the use of the California relation between fat and solids-not-fat. The difference between this and the basic California relation reflects the influence of the fat carried over into the skim milk and the allowance made for the physical losses in separation. It is further assumed that processing losses in the drying operation are exactly compensated for by moisture retention.

The net values of milk used for butter and powder, therefore, can be expressed:

$$V_m = (1.23F - .123)(P_b - C_b) + (7.17 + .441F)(P_{nfs} - C_{nfs}) - C_{rs}$$

where V_m is the net value of whole milk per hundred pounds; F is again the milk fat test; P_b the price of butter; C_b the direct costs per pound of processing butter, including such marketing costs as are necessary to bring the butter to the point of sale; P_{nfs} the price of nonfat dried solids; C_{nfs} the direct costs per pound of processing and marketing nonfat solids; and C_{rs} the costs per hundredweight of receiving and separating the whole milk, which are not included in the direct production costs.

The butterfat differential—or the difference in net value of whole milk for each change of $\frac{1}{10}$ of 1 per cent in fat test—can also be computed directly. The formula for such computation will be:

$$BFD_{bp} = .123(P_b - C_b) + .044(P_{nfs} - C_{nfs})$$

where BFD_{bp} refers to the butterfat differential appropriate for a butter-powder operation, in dollars per hundredweight per $\frac{1}{10}$ of 1 per cent change in fat test; the expression $(P_b - C_b)$ reflecting the net value per pound—price less processing and marketing costs—of butter; and $(P_{nfs} - C_{nfs})$ the net value per pound of dried nonfat solids.

A "computing table" has been set up in table 2 which indicates how the above equations might be used to determine values for any specific time period. To do this, of course, it is necessary to apply appropriate cost and price figures, which change from one period to another. It is essential, therefore, that the cost and price data used be those which relate to the time period for which the values are to be calculated. In this example, the following have been used: price of butter, per pound (P_b), \$0.685; price of nonfat solids (P_{nfs}), \$0.18 per pound; direct processing and marketing costs for butter (C_b), \$0.05 per pound; direct processing and marketing costs for dried nonfat solids (C_{nfs}), \$0.07 per pound; and the receiving and separating cost, \$0.20 per hundredweight of whole milk. It will be further assumed that milk of 4.0 per cent fat content is to be used.

Evaporated Milk Operations.— Milk to be processed into unsweetened, unskimmed, evaporated milk is typically standardized to a desired ratio of fat and nonfat solids before being concentrated since these ratios of fat and nonfat solids vary, as do the absolute quantities present. Milk with low fat test contains a high proportion of nonfat solids relative to fat, and, conversely, high fat milk contains a high percentage of fat in relation to the nonfat solids. Milk of 3.9 per cent fat content contains these solids in approximately the proportion that requires no standardization, according to the California relationship of fat to solids-not-fat.[17] For this reason, an evaporated milk operation will be accompanied by a by-product operation whenever the average test of incoming whole

[17] According to the standards used, and the level of nonfat solids indicated by the California relation, the test of milk is normally in the correct ratio at 3.855 per cent fat. For simplicity of statement, this is rounded to 3.9 per cent.

milk differs from 3.9 per cent. It is, for the purpose of this example, assumed that milk of lower test will be standardized by the removal of skim milk which, in turn, will be processed into nonfat dried milk solids. Milk of a higher average test than 3.9 will be standardized by the removal of 40 per cent cream which will be converted into butter with the buttermilk yield omitted because of its negligible size. Under such conditions, the following product yields will result:

(a) for milk with less than 3.9 per cent fat
$Q_e = .291F - .023$ and
$Q_{nfs} = 7.14 - 1.85F$

(b) for milk with more than 3.9 per cent fat
$Q_e = .013F + 1.05$
$Q_b = 1.24F - 4.78$

where Q_e is number of cases (forty-eight 14½-ounce cans) of evaporated milk per 100 pounds of milk; F the fat test of milk; Q_{nfs} the pounds of dried nonfat solids; and Q_b the pounds of butter obtained per hundredweight of milk, respectively.

Due to the differences in the physical yield relationships, it is necessary to establish two expressions to determine the net value of milk used to manufacture evaporated milk—one for milk above and one for milk below the basic 3.9 per cent fat content. These are:

(a) for milk of less than 3.9 per cent fat
$V_m = (.291F - .023)(P_e - C_e) + (7.14 - 1.85F)(P_{nfs} - C_{nfs}) - C_{rs}$

(b) for milk of more than 3.9 per cent fat
$V_m = (.013F + 1.05)(P_e - C_e) + (1.24F - 4.78)(P_b - C_b) - C_{rs}$

where V_m is the net value of milk used for evaporated milk; the expressions $(P_x - C_x)$ reflect the net value per unit—price less processing and appropriate

Table 2. Calculating Table to Determine "Net Values" of Milk When Used in a Butter-Powder Operation

Item	Symbol	Calculation
1. Yield of butter, pounds	$(1.23F - .123)$	$1.230 \times 4.000 - .123 = 4.797$
2. Net value of butter, per pound	$(P_b - C_b)$	$.685 - .050 = .635$
3. Total net value of butter	item 1 × item 2	$4.797 \times .635 = 3.046$
4. Yield of dried nonfat solids, per pound	$(7.17 + .441F)$	$7.170 + .441 \times 4.0 = 8.934$
5. Net value of dried nonfat solids	$(P_{nfs} - C_{nfs})$	$.180 - .070 = .110$
6. Total net value of dried nonfat solids	item 4 × item 5	$8.934 \times .110 = .983$
7. Net value of butter plus powder	item 3 + item 6	$3.046 + .983 = 4.029$
8. Receiving and separating cost	(C_{rs})	$.200 = .200$
9. Net value of whole milk	item 7 − item 8	$4.029 - .200 = 3.829$
10. Butterfat differential*	.123 (item 2) + .044 (item 5)	$.123 \times .635 + .044 \times .11 = .0829$

* The corresponding "direct-ratio-to-fat" butterfat differential would amount to $.0957 for each 1/10 of 1 per cent difference in fat content per hundredweight of milk.

Table 3. Calculating Table to Determine "Net Values" of Milk When Used in an Evaporated Milk Operation

Item	Symbol	Calculation
1. Yield of evaporated milk, cases	$.013F + 1.05$	$.0130 \times 4.000 + 1.05 = 1.102$
2. Net value of evaporated milk, per case	$(P_e - C_e)$	$6.2400 - 2.050 = 4.190$
3. Total net value of evaporated milk	item 1 × item 2	$1.1020 \times 4.190 = 4.617$
4. Yield of butter	$1.24F - 4.78$	$1.2400 \times 4.000 - 4.78 = .180$
5. Net value of butter, per pound	$(P_b - C_b)$	$.6850 - .050 = .635$
6. Total net value of butter	item 4 × item 5	$.1800 \times .635 = .114$
7. Net value of evaporated milk plus butter	item 3 + item 6	$4.6170 + .114 = 4.731$
8. Receiving and separating cost	(C_{rs})	$.200 = .200$
9. Net value of whole milk	item 7 − item 8	$4.7310 - .200 = 4.531$
10. Butterfat differential*	.0013 (item 3) + .124 (item 5)	$.0013 \times 4.617 + 1.24 \times .635 = .0847$

* The corresponding "direct-ratio-to-fat" butterfat differential would amount to $.1133 for each 1/10 of 1 per cent differential in fat content per hundredweight of milk.

marketing costs—of the respective products where the subscript e refers to cases of evaporated milk, nfs to pounds of dried nonfat solids, and b to butter in pounds; and where C_{rs} refers to the joint costs of receiving and separating whole milk.

In this instance, the butterfat differentials which would directly express the difference in net values per $\frac{1}{10}$ of 1 per cent change in fat content per hundredweight of milk are:

(a) for milk of less than 3.9 per cent fat

$$BFD_e = .029\ (P_e - C_e) - .185\ (P_{nfs} - C_{nfs})$$

(b) for milk of more than 3.9 per cent fat

$$BFD_e = .0013\ (P_e - C_e) + .124\ (P_b - C_b)$$

where BFD_e refers to the butterfat differential in dollars per 100 pounds of milk appropriate for a plant primarily manufacturing evaporated milk; and the parenthetical expressions again reflect the net values per unit of the respective products.

An example showing a set of calculations using these relations, similar to those shown in table 2, is presented for a condensery operation. These appear in table 3, and involve the following assumed prices and costs: evaporated milk prices per case (P_e), $6.24; dried nonfat solids price per pound (P_{nfs}), $0.18; butter price per pound (P_b), $0.685; processing and marketing costs of evaporated milk per case (C_e), $2.05; processing and marketing costs of butter per pound (C_b), $0.05; processing and marketing costs of dried nonfat solids (C_{nfs}), $0.07; and the cost of receiving and separating whole milk per hundredweight (C_{rs}), $0.20. As with the example in table 2, it will be assumed that milk of 4.0 per cent fat content is used.

Fluid Cream and Skim Milk Operations.—Another alternative outlet for manufacturing grade milk, by California

plants, involves the sale or use of fluid cream and skim milk for other manufactured dairy products, especially for ice cream and cottage cheese. Following similar procedures to those outlined above, the net value of milk going into such operations can be calculated. The quantities of fluid cream and fluid skim milk that can be obtained from a hundredweight of milk are:

$$Q_{40} = 2.48F - .248 \text{ and}$$
$$Q_s = 99.248 - 2.48F$$

where Q_{40} refers to the number of pounds of cream of 40 per cent fat content, Q_s, the number of pounds of fluid skim milk, and F, the fat test of milk being processed.

The net value of milk for these purposes can again be stated as the quantity of product valued at the current net prices (or values) per pound. This equation is:

$$V_m = (2.48F - .248)\ (P_{40} - C_{40}) + (99.248 - 2.48F)\ (P_s - C_s) - C_{rs}$$

where V_m is the net value of milk used for separation into fluid cream and skim milk for sale or use in these other manufactured dairy products, P_{40} and P_s refer to the prices of 40 per cent cream and skim milk, respectively, C_{40} and C_s are the direct costs (if any) of processing and marketing cream and skim milk, respectively, and C_{rs}, the costs of receiving and separating the whole milk.

The butterfat differential appropriate for an operation of this type may be expressed:

$$BFD_{cs} = .248\,[\,(P_{40} - C_{40}) - (P_s - C_s)\,]$$

where BFD_{cs} refers to the butterfat differential in dollars per hundredweight of milk used in a cream-skim milk operation, per $\frac{1}{10}$ of 1 per cent change in fat test, and where the parenthetical expressions relate to net prices or values for 40 per cent cream and skim milk, respectively.

A "calculating table" illustrating the application of the above equations relating to a manufacturing grade fluid cream and skim milk operation appears in table 4. For the purpose of this example, the following prices and costs are used: the price of manufacturing grade cream of 40 per cent fat content per pound of cream (P_{40}), \$0.364; the price of manufacturing grade skim milk per pound of skim milk (P_s), \$0.012; the direct costs of processing and marketing the cream and skim milk (C_{40}) and (C_s) are both considered to be zero; and the cost of receiving and separating whole milk (C_{rs}), \$0.20 per hundredweight. Again it will be assumed that milk of 4.0 per cent fat content is entering the plant.

Market Milk Prices.—Although the variation in fat and nonfat solids content is as great in market milk as in manufacturing grade milk, the problem of evaluating the separate components in fluid use becomes more complex. For one thing, it is difficult to determine the influence of different amounts of nonfat solids on the value of milk used for fluid purposes. While nutritional differences do exist, most consumers would find it difficult to recognize such variations as are normally found in the nonfat solids content of the milk they buy. Unless thoroughly convinced of the differences in nutritional benefits involved, and at the same time advised of the variation in the content of solids of this type, it is unlikely that consumers would be willing to pay a premium for relatively high nonfat solids in fluid milk. This being the case, the variation in nonfat solids content is not a satisfactory basis for making direct determination of the difference in value of different types of milk when used for fluid purposes.[18] The following discussion, therefore, is not concerned

[18] It should be noted that increased emphasis on nonfat solids, in the future, may make it desirable to include price variations as a result of the difference in the nonfat solids content of the skim milk when used for fluid purposes. Some tendency in this direction is indicated by the fact that some fluid skim milk sold to consumers is enriched with added nonfat solids. In addition, others—particularly Professor Misner

with the differences in the solids content of the skim milk.

For practical purposes, the components of market milk are cream and fluid skim milk. Under California pricing and sanitary regulations, cream and fluid skim milk made from Grade A milk may have a higher value than physically identical cream and skim milk derived from milk of manufacturing grade quality. This is due to the fact that such Grade A cream and skim milk can be combined into products such as market milk, cream, and half-and-half, which yield a greater net return per hundredweight of raw product than do butter, evaporated milk, and the other manufactured dairy products.[19] The difference in value between market milk and manufacturing milk of similar physical characteristics may be called the "Grade A premium" or, in cases where the two terms are not synonymous (as they are in California), the "Class I premium."

The amounts of cream and skim milk per hundredweight of whole milk are, of course, directly related to the amount of fat present in the original milk. For example, approximately 7.2 pounds of 40 per cent cream and 91.8 pounds of skim can be obtained from 100 pounds of 3 per cent whole milk, while roughly 12.2 pounds of 40 per cent cream and 86.8 pounds of skim will be produced from

Table 4. Calculating Table to Determine "Net Values" of Manufacturing Grade Milk When Used in a Fluid Cream and Skim Milk Operation

Item	Symbol	Calculation
1. Yield of cream, pounds	$2.48F - .248$	$2.480 \times 4.000 - .248 = 9.6720$
2. Net value of cream, per pound	$(P_{40} - C_{40})$	$.364 - .000 = .3640$
3. Total net value of cream	item 1 × item 2	$9.672 \times .364 = 3.5210$
4. Yield of skim milk, pounds	$99.248 - 2.48F$	$99.248 - 2.480 \times 4.000 = 89.3280$
5. Net value of skim milk, per pound	$(P_s - C_s)$	$.012 - .000 = .0120$
6. Total net value of skim milk	item 4 × item 5	$89.328 \times .012 = 1.0720$
7. Net value of cream plus skim milk	item 3 + item 6	$3.521 + 1.072 = 4.5930$
8. Receiving and separating cost	(C_{rs})	$.200 = .200$
9. Net value of whole milk	item 7 − item 8	$4.593 - .200 = 4.3930$
10. Butterfat differential*	.248 (item 2 − item 5)	$.248\ (.364 - .012) = .0873$

* The corresponding "direct-ratio-to-fat" butterfat differential would amount to $.1098 for each 1/10 of 1 per cent difference in fat content per hundredweight of milk.

of Cornell University—have emphasized that milk could be priced according to energy values as expressed in terms of calories. The point to be made here is that modifications can be adapted to the material presented here to incorporate these or other procedures when it appears desirable. As an illustration, specific formulas which will account for differences in the nonfat solids content of milk when used for fluid purposes appear in Appendix B.

[19] Where a "classified price plan" is used—as in California and commonly in many other milk markets throughout the country—a price premium is paid to producers for milk which goes into fluid uses. Under California sanitary regulations, these higher priced fluid uses require the use of Grade A milk, although this latter provision is not universally held in other markets.

the same amount of 5 per cent milk.[20] The higher the fat content of the original milk, the greater is the amount of cream that can be derived per 100 pounds of milk and, conversely, the smaller the amount of residual skim milk.

In California, any Class I milk product—such as market milk, market cream, and fluid skim milk—can be considered as a combination of Grade A fat and Grade A skim milk, provided that the solids content of the skim milk meets the minimum requirement of 8.15 per cent.[21] The value of milk of this quality to a distributor is the combined value of the quantities of fat and of skim milk which it will produce. The major difficulty in formulating specific pricing schedules arises from attempting to determine the appropriate values of these separate components in market grade milk. To be sure, these are at least as valuable to any given plant as the same components derived from manufacturing grade milk for, lacking a more profitable use, market grade fat and skim milk can be diverted into butter, cheese, and other dairy products. As mentioned earlier, however, when used for specific fluid purposes, "Class I" milk receives a price premium. The problem of pricing fluid milk with different fat and skim yields includes the allocation of the Class I premium between the two components.

Like many accounting problems which involve the allocation of common or "joint" costs, there is no single "correct" procedure for the allocation of the Class I premium to the skim and to the fat constituents of market milk. Possibly a "reasonable" basis for allocating the Grade A premium *could* be determined that would be acceptable to all. Nevertheless, there is no "correct" method or single "right answer" to this problem.

There is little information available through regularly reported statistics from which indications of the market valuations of this Class I premium on the separate components of milk can be drawn. Prices for market grade cream and market grade skim milk (which in California are Grade A) have been collected for the period of June, 1950, to May, 1951, as a part of a previously reported study.[22] These prices represented interarea, interdistributor sales, and were not subject to control by the price-fixing agency of the state. The ranges of these prices are shown in table 5. As can be seen, there was considerable variation between the prices for any given month at which these physically and legally identical commodities were sold. Because these prices are not subject to control, they may provide an indication of the way the market tends to operate in allocating these premiums. If it can be safely assumed that the higher prices are those paid for supplies going into Class I uses, while the lower range of prices relates to "surplus" diverted into manufactured products, then the difference between the two would be a reflection of the Class I premium on the one hand for the fat-containing product and on the other, for fluid skim milk. It should be noted that the high prices of the reported range conform closely to the price quotations of the Grade A jobbing cream market, while the low prices in general reflect the manufacturing grade cream quotations.

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[20] The theoretical yield of 40 per cent cream would be two and one-half times the butterfat content. However, milk which has been skimmed by commercial methods normally contains at least 1/10 of 1 per cent milk fat and, for this reason, the actual yield of cream is slightly below the theoretical level—roughly one-fourth pound per hundredweight of milk. In addition, there is normally a product loss of about 1 per cent in separating so that the cream and skim milk will total only 99 pounds per hundredweight of milk.

[21] According to the California fat-nonfat solids relationship, milk with an average of 2.5 per cent fat or above will have sufficient nonfat solids to meet this requirement.

[22] "Pricing Intermarket Transfers of Bulk Grade A Cream and Skim Milk," by D. A. Clarke, Jr. Calif. Agr. Exp. Sta. Bul. 732, 1952.

Table 5. The Range of Selling Prices Reported for Interplant Shipments of Grade A Cream and Skim Milk, June, 1950–May, 1951

Month	Cream prices per pound of fat		Skim milk prices per hundredweight	
	Low	High	Low	High
	dollars			
1950:				
June	0.75	1.00	1.24	1.40
July	0.76	1.00	1.23	1.44
August	0.77	1.00	1.25	1.40
September	0.83	1.04	1.25	1.72
October	0.84	1.06	1.25	2.33
November	0.84	1.04	1.25	2.33
December	0.82	1.06	1.25	1.80
1951:				
January	1.04	1.16	1.36	1.80
February	1.04	1.16	1.38	1.80
March	0.96	1.17	1.45	1.80
April	0.96	1.09	1.47	1.80
May	0.96	1.10	1.47	1.80

Source: Calif. Agr. Exp. Sta. Bul. 732, p. 21.

In addition to the problem of determining an "equitable" basis for the allocation of the Class I premium between the fat and skim components of market grade milk, there is the procedure for determining class usage and, consequently, the prices paid to producers. As previously mentioned, fluid milk in California is paid for according to a classified price plan. Certain uses, such as fluid milk, concentrated milk, fluid cream, fluid skim milk, and chocolate drink, are defined by the Agricultural Code as Class I. Thus, milk used for these purposes is paid for at the Class I prices as established by the Bureau. Market grade milk in excess of these requirements, so-called "surplus," is normally used for manufactured products where the law does not require Grade A standards. Milk used for these purposes is paid for at Class II and III prices. These are also established by the Bureau, but at lower levels than the Class I price and normally in close relationship with prices for manufacturing grade milk.[23] In spite of the fact that some of the Class I products are primarily fat containing, such as cream, and some predominantly skim containing, like fluid skim milk and chocolate drink, and, furthermore, that fluid milk is normally standardized to a different level of fat and skim proportions than exist in whole milk received from producers, the prices paid producers under present California procedures depend solely on the class use of fat.

Under these circumstances, a distributor with a high Class I skim utilization relative to Class I sales of fat pays Class

[23] While the Agricultural Code makes provision for separate prices for Class II and III uses, where Class III includes milk used for butter and cheese other than cottage cheese, and where Class II includes all uses other than those defined as Class I or III, the Class II and III prices established by the Bureau quite often are identical.

II or III prices for the skim milk in excess of that associated with the fat accounted for at Class I prices. These Class II and III prices are closely related to, and in many markets identical with, manufacturing grade milk prices. Conversely, of course, a distributor with relatively high Class I fat utilization effectively pays a Class I premium for more skim milk than is used in Class I products when these supplies are obtained from whole milk received from producers.

As a result of the inequities inherent in this procedure of accounting for usage, it has been suggested that *separate prices* be established for both the fat and skim milk components of market grade milk.[24] These separate prices would then apply to the actual utilization of whole milk in both the fat-containing and skim-containing products, and would require that distributors further account for skim usage.

Schedules of prices to be paid producers for milk of varying fat content have been established in milk markets throughout the United States. These schedules—whether established by public agencies, market-wide producer-distributor negotiations, or by a single company—are generally of two types, namely, (1) a hundredweight price schedule or (2) a constant price per pound of fat in the whole milk with no direct value being placed on the skim milk. The hundredweight schedule is the series of prices for 100 pounds of whole milk with different fat content. This schedule is usually postulated by means of a *base* price for milk having the prescribed basic fat test (3.8 per cent in California markets) in conjunction with a specific butterfat differential. Consequently, such pricing schedules are linear.

The second pricing schedule listed above is usually called "direct ratio" pricing. On a hundredweight basis, milk of various fat tests, when priced in this manner, has values in direct ratios to the fat test. Direct ratio pricing is equivalent to a hundredweight schedule with a butterfat differential equal to $1/10$ of the price per pound of fat.

The reason for the discussion of whole milk paying schedules is to point out now that such schedules determine important relationships which must be understood if (under classified pricing) a system of accounting to the producer on the basis of both fat and skim utilization is to be internally consistent. The development to follow implies that skim milk used in Class I purposes is homogeneous in value, that is, variations in normal solids content have no evident influence on value. With this assumption, the base price per hundredweight for milk ($P_F{}^*$) would be represented by:

$$P_F{}^* = F^* P_f + (100 - F^*)\, P_s$$

or the value of F^* pounds of fat at price P_f plus the value of the remaining skim $(100 - F^*)$ at the price P_s for skim.[25] In addition, the butterfat differential would be

$$BFD = 0.1 P_f - 0.1 P_s$$

This means that each additional 0.1 pound of fat will mean a reduction of 0.1 pound of skim milk, and that the butterfat differential is simply the "net" increase in value, or the value of 0.1 pound of fat less the value of 0.1 pound of skim milk.

It should be noted from the above relationships among the four defined variables that the specification of the values of any two of them automatically determines the consistent values for the re-

[24] Calif. Agr. Exp. Sta. Bul. 732, p. 26.

[25] Fat, in this sense, refers specifically to the "pure" fat component, and the price for this should not be confused with the price "per pound of fat" often quoted for whole milk and for cream—both of which contain quantities of skim milk. Skim milk, in this sense, is not the common product called "skim milk" which contains a small portion of residual milk fat, but is a "fat free" component of whole milk.

maining two. For example, if F^* is 3.8, $P_F{}^*$ is \$5.46, and BFD is .10, then, upon substitution, we have

$$3.8P_f + 96.2P_s = 5.46$$
$$.1P_f - \quad .1P_s = \; .10$$

Upon solving these equations, one secures $P_f = \$1.02$ and $P_s = \$.0166$.

It is now apparent that any butterfat differential, given a base milk price, has an intimate connection with the consistent fat and skim values in the whole milk, and that the various fat and skim prices associated with various butterfat differentials will have marked differences of economic significance. This immediately suggests that one should seek in the milk marketing system some economic indicators of market evaluations of fat and skim milk to be used for the determination of the butterfat differential, instead of employing some arbitrary butterfat differential and changing it later when the "consistent" fat and skim milk prices are not in accord with market evaluations. The subsequent pages of this section will be devoted to a general discussion of some approaches to the problem of estimating Class I fat and skim milk values, and a more detailed presentation of these approaches will be given in a later section.

GUIDES FOR DETERMINING SEPARATE PRICES FOR FAT AND SKIM MILK IN CLASS I WHOLE MILK

Froker and Hardin[26] have suggested that this problem might be solved on an "alternative value" basis. In this case, values would be assigned to market grade fat and market grade skim milk in proportion to the relative values of these components in a butter-powder operation. In other words, the net value of whole milk of specified fat content is computed on the basis of current market values of butter and nonfat dried milk solids. From this, the *percentage* contributions of the fat and the skim milk are calculated, the sum of which, of course, amounts to 100 per cent. When these percentages are applied to a *given* Class I price for milk of the same test, the resultants are taken to be the values of the market grade fat and the skim milk, respectively. As a simple illustration, assume that the net value of butter at current levels of prices and costs is 75 per cent of the net value of whole milk of 3.8 per cent fat content used for butter and powder. Seventy-five per cent of the established Class I price of 3.8 milk would then represent the value of 3.8 pounds of fat and, similarly, the remaining 25 per cent of the price of whole milk would represent the value of the residual 96.2 pounds of skim milk. In this way, the relative values of fat and skim milk for fluid uses are determined by the relative values of these components in the butter and nonfat dried milk solids markets.

The advantages of this procedure appear to be twofold. In the first place, it is relatively familiar, and may be easily understood by the dairy industry. In the second place, it involves prices—those for butter and for nonfat dried milk solids—which are widely and currently available. The primary disadvantage is that there is no logical reason why the fat and skim components of market grade milk either would or should bear the same relation as those established by the butter and the dried milk solids market, particularly when the market prices for these products are artificially maintained at arbitrary levels under the government support program.

An alternative procedure involves a second major premise—that current

[26] See footnote 13.

market grade jobbing cream price quotations provide a "reasonable" guide to market valuations of market grade fat. This argument proceeds logically on the grounds that the only physical or legal difference between this cream and the whole milk from which it was derived is that of the relative proportions of fat and skim milk contained therein. The Grade A jobbing cream prices in California are "open market" prices, that is, not fixed by administrative order. Under such circumstances, and where distributors are free to obtain market grade fat and skim milk supplies either in the form of cream or as whole milk, it may be claimed that the relative prices of these alternatives reflect the way the market evaluates the *difference* in constituents.

The major advantage to be claimed for this alternative procedure is that it relates prices for Class I fat and Class I skim milk directly to the operation of Class I markets. In view of the classified price system used in California, and in many other milk markets, together with the existence of institutional "barriers," such as geographical supply area limitations and quotas, which prevent the free flow of milk between uses (particularly into the higher value, or Class I, uses), the relative prices derived from manufacturing operations are not necessarily, and in fact would not be expected to be, good indicators of those resulting from fluid milk market operations.

The major criticisms of the use of this procedure are based on the following arguments. First, that the California Grade A jobbing cream market prices are not independent of the Class I whole milk prices but are, in fact, closely related, so that it may be argued that the Class I prices "set" the Grade A jobbing cream prices. Second, that there are relatively small sales made on the jobbing cream market, so that quoted prices have questionable significance — particularly in certain periods of the year.

The first of these objections—that concerning the relationship between these prices—is not a major limitation, but one of the primary bases for suggesting this procedure. As stressed previously, if the Grade A jobbing cream market operates effectively, then it will tend—within the whole milk price structure—to value the fat and skim milk when in different combinations from those existing in whole milk and, by the difference between prices, to value the difference in Grade A skim milk content between a given quantity of fat in whole milk and in cream.

The second criticism—that insufficient quantities of cream are sold in the jobbing market to enable establishment of "true" values—is more difficult to evaluate. For many commodities, however, prices are established on the basis of the operation of produce exchanges and produce auctions. Not infrequently, the quantities which actually pass through these markets are but a very minor proportion of the total quantities sold. Yet, it is argued that these markets—in the absence of possibilities for control or "rigging"—are sufficiently active to provide a focal point for the operation of supply and demand. It should be further remembered that it is not suggested that the jobbing cream prices be used to establish the level of whole milk prices. Rather, these cream prices could be used to determine the allocation of the whole milk price between the fat and skim milk constituents and thus to determine the prices to be paid to producers for milk of different constituencies to be used for Class I purposes. Should the market for jobbing cream operate ineffectively, distributors would find that one of the constituents was consistently overpriced while, at the same time, the other component was consistently underpriced, relative to values. It is felt that this situation, continuing over any length of time, would automatically set forces in motion which would, in turn, be reflected in the jobbing cream market in a corrective manner.

No attempt is made in this report to develop a formula or equation which will determine the *level* of Grade A milk prices. Rather, as mentioned earlier, it is assumed that these prices will continue to be established, as currently, by the Bureau of Milk Control. It is useful, however, to establish the following relationship between the price for whole milk and the prices for the basic constituents of that milk. The following basic assumption will be followed throughout the later discussion:

$$P_m = V_f + V_s$$

which states that the price of milk is equal to the value of the fat and skim constituents contained therein. When market or Class I milk is concerned, the above must be modified to include the value of the Class I premium and its allocation between the fat and skim milk under existing market conditions. Alternative procedures—both of which are by necessity arbitrary — are presented which would accomplish the purpose of allocating this premium.[27]

Procedure 1. This method is based on the assumption that the relative values of fat and skim milk used for Class I purposes will be reasonably reflected by the relative values established by the butter and dried nonfat solids markets. Initially, the net value of milk in a butter and powder operation can be calculated in the manner previously discussed. This involves the basic formula:

$$V_m = (1.23F - .123)\,(P_b - C_b) + (7.17 + .441F)\,(P_{nfs} - C_{nfs}) - C_{rs}$$

where V_m is the net value of whole milk per hundred pounds; F, the milk fat test; P_b, the price of butter; C_b, the direct costs of processing butter, including such marketing costs necessary to bring the butter to the point of sale; P_{nfs}, the price of nonfat dried solids; C_{nfs}, the direct cost of processing and marketing nonfat solids; and C_{rs}, the costs of receiving and separating whole milk.

If the joint receiving and separating costs are allocated according to the relative values of fat and skim in the whole milk prior to the subtraction of these joint costs, then the percentage contribution of the fat component to the whole milk value is given by:

$$RVF_F = \frac{(1.23F - .123)\,(P_b - C_b)}{V_m + C_{rs}}$$

where RVF_F indicates the relative value of fat in whole milk with fat content of F per cent, and the other subscripts have the meanings indicated above. It follows that the percentage contribution of the nonfat solids to the whole milk value would be:

$$RVNFS_F = 1 - RVF_F$$

The value of RVF_F (and also $RVNFS_F$) is not constant for all values of F, but varies with the fat test of the whole milk under consideration. Consequently, when applying the relative value concept to the Class I milk, RVF_F must be computed for that value of F which is *the basic fat test of whole milk* for which the Class I price has been established.[28]

[27] It should be noted that these two methods do not exhaust all possible procedures, but are merely presented as illustrations of logical methods. For example, other procedures might be adopted, similar to those presented for manufacturing grade milk prices, involving "net value" formulations based on retail prices for fluid products less processing costs. However, these have not been discussed for fluid operations in detail, due primarily to the nonhomogeneity of product utilization in fluid milk operations and the corresponding differences in realized net product prices for the variety of fluid milk and cream items.

[28] The fact that these relative values change for different levels of fat content can be demonstrated through the following:

$$RVF_F = \frac{1}{1 + \left(\frac{P_{nfs} - C_{nfs}}{P_b - C_b}\right)\left(\frac{7.17 + .441F}{1.23F - .123}\right)}$$

For any given set of prices and costs, the first set of terms in parentheses—reflecting net product prices—will be constant. However, the second set—those which reflect the product yields—is not constant, but is functionally related to the fat content, expressed here as F. For example, for 3 per cent milk, this second term has a value of approximately 2.4, while for milk of 6 per cent fat content, this paren-

The value that results when the basic Class I price is multiplied by RVF_F, can be taken as the Class I value of the amount of fat present in the milk. To obtain the value of a pound of fat, it is only necessary to divide through by the fat test. Similarly, the difference between the value of fat and the Class I price reflects the skim milk value, or the value of the amount of skim milk in 100 pounds of whole milk. Assuming the situation where the basic fat test is 3.8 per cent, the above results can be stated by the following equations:

$$\text{(a)}\ VF_{Cl.\,I} = \frac{(P_{3.8})\,(RVF_{3.8})}{3.8} \text{ and}$$

$$\text{(b)}\ VS_{Cl.\,I} = \frac{(P_{3.8} - VF_{Cl.\,I})}{96.2}$$

where $VF_{Cl.\,I}$ is the Class I value of a pound of milk fat; $P_{3.8}$, the hundredweight price of whole milk of 3.8 per cent fat—as independently established; $RVF_{3.8}$, the relative value of fat as determined for F equal to 3.8 from the butter powder calculations; and $VS_{Cl.\,I}$, the value of a pound of skim milk in whole milk having a fat content of 3.8 per cent. The butterfat differential would be given by: $BFD = 0.1VF_{Cl.\,I} - 0.1\ VS_{Cl.\,I}$.

As an example to illustrate this procedure, assume that the current price of butter is \$0.685 per pound; the price of dried nonfat solids is \$0.18 per pound; the direct costs of processing and marketing butter and dried nonfat solids are \$0.05 and \$0.07 per pound of product, respectively; the joint costs of receiving and separating whole milk are \$0.20 per hundredweight; and that the current price for Class I milk, as established by the administrative agency for milk of 3.8 per cent fat content, is \$5.46. The first step involves substituting the appropriate prices, fat tests, and costs in order to determine the total net value of milk of 3.8 per cent fat content into the "net value" equation as follows:

$$V_m = (1.230 \times 3.8 - .123)\ (.685 - .05) + (7.17 + .441 \times 3.8)\ (.18 - .07) - .20$$

$$V_m = (4.551)\ (0.635) + (8.846)\ (.11) - .20$$

$$V_m = 2.890 + .973 - .20$$

$$V_m = 3.663$$

which represents the total net value, or "paying ability" of milk of 3.8 per cent fat content used in manufacturing butter and powder under the above costs and prices.

To determine the relative value of the fat in this milk, it is necessary to divide the contribution of the butter (which is taken to represent only the fat element) by the total value of the milk. This is accomplished in the following manner:

$$RVF_m = \frac{2.890}{3.663 + .20}$$

$$RVF_m = .748$$

where, in the above, it has been further assumed that the joint costs of receiving and separating the whole milk (\$0.20) will be allocated between the fat and skim milk in accordance with the relative values. As shown above, approximately 75 per cent of the value of milk in a butter-powder operation is associated with the fat. Since the theory behind this procedure involves the application of the relative value of fat in the butter-powder operation to determine the value of fat in Class I milk, the next step is the following:

$$P_{3.8}(RVF_{3.8}) = \$5.46 \times .748$$

$$P_{3.8}(RVF_{3.8}) = \$4.084$$

which may be taken to represent the value of the 3.8 pounds of fat in a hundredweight of 3.8 per cent milk. The value per pound of fat would then be:

$$VF_{Cl.\,I} = \frac{\$4.084}{3.8}$$

$$VF_{Cl.\,I} = \$1.075$$

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thetical term has a value of about 1.4. This results from the fact that butter yields increase more rapidly than do yields of dried nonfat solids as the fat content of milk increases. As a consequence, the relative value of fat (RVF_F) increases as the level of fat content of milk increases.

As the value of skim represents the difference between the value of whole milk and the fat value, the skim milk contained in 100 pounds of 3.8 milk (96.2 pounds) may be determined:

$$96.2\ VS_{Cl.\ I} = \$5.46 - \$4.084$$
$$VS_{Cl.\ I} = \$0.0143$$

The value of the butterfat differential, consistent with these separate prices for fat and skim milk, would be:

$$BFD = 0.1\ (1.075) - 0.1\ (.0143)$$
$$BFD = .1075 - .0014$$
$$BFD = .1061$$

This compares with a "direct-ratio-to-fat" butterfat differential of .1437 for each 1/10 of 1 per cent difference in fat content per hundredweight of milk.

Procedure 2. The second alternative for the determination of the separate Class I values of fat and skim milk is based on the assumption that the Grade A jobbing cream market in California provides a reasonable basis for determining the way in which the market values the individual components of market grade milk.

It should be recognized that the jobbing cream price is at a different level in the marketing system from that of the established base price for whole milk. The whole milk price is established for milk entering a plant, while the cream price is associated with a milk product that is leaving a plant and has undergone physical loss and processing costs. In order to use these two prices to determine the values of fat and skim milk in Class I milk, it is necessary to adjust the cream price back to a "plant entry" value. This requires the subtraction of the direct processing and marketing costs for cream and an allocated share of the general receiving and separating costs as well as an adjustment for the physical loss.

As stated previously, the quantities of 40 per cent cream (which correspond to the Grade A jobbing cream price quotations) and of skim milk that are obtainable from 100 pounds of milk are:

$$Q_{40} = 2.48F - .248\text{, and}$$
$$Q_s = 99.248 - 2.47F$$

Since the prices that are to be used for whole milk are those established by the Bureau of Milk Control at the basic fat test of 3.8 per cent, this latter fat test can be substituted, in the above equation, for F. Making this substitution, the equation results in estimates of 9.2 pounds of 40 per cent cream and 89.8 pounds of skim milk per hundredweight of 3.8 per cent milk.[29]

Jobbing prices for 40 per cent cream are usually quoted on a per pound fat basis. It will be convenient to consider the 9.2 pounds of 40 per cent cream as being equal to 3.68 pounds of fat and 5.52 pounds of skim milk. Consequently, 3.68 times the fat price for 40 per cent cream will include the value of 5.52 pounds of skim milk, or equivalently, the fat price for 40 per cent cream is the value of 1 pound of fat and 1.5 pounds of skim milk.

The cream price adjusted to a "plant entry" basis would be:

$$^{*}V_{40} = (0.99)\ (P_{40} - C_{40} - {}^{*}C_{rs})$$

where $^{*}V_{40}$ is the per pound fat price for 40 per cent cream at the "plant entry" level, 0.99 is the physical loss conversion factor, P_{40} is the per pound fat price for 40 per cent cream for jobbing sales, C_{40} is the direct processing and marketing costs for cream expressed on a per pound fat basis, and $^{*}C_{rs}$ is the share of joint receiving and separating costs of whole milk which have been allocated to the cream on a per pound fat basis.[30] It is

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[29] The fact that the two quantities of product do not add to 100 pounds reflects the allowances that have been made for product loss during separation and handling. As previously stated, the allowances include a 1/10 per cent fat loss to the skim milk, and a 1 per cent loss in total product in receiving and separating.

[30] $^{*}C_{rs}$ must, of necessity, be an arbitrary allocation. If one wishes to allocate the receiving and separating costs to cream and the separated skim milk on a relative net value basis, then $^{*}V_{40} = (0.99)\ (P_{40} - C_{40}) \frac{P_{3.8}}{P_{3.8} + C_{rs}}$, where C_{rs} is the total receiving and separating costs per 100 pounds of whole milk.

evident that one must have values for C_{40} and $*C_{rs}$ in order to adjust a specific value of P_{40}.

The solution for the basic fat and skim milk values in the whole milk is easily accomplished with the knowledge of the base price for whole milk and the above adjusted 40 per cent cream value. In fact, the following equalities must hold, and their simultaneous solution gives the consistent fat and skim milk values:[31]

$$3.8\,P_f + 96.2\,P_s = P_{3.8}$$
$$P_f + 1.5 P_s = *V_{40}$$

where P_f and P_s are, respectively, the per pound fat and skim values (prices) in whole milk, and the other symbols have been previously defined.

The solution of the above system results in

$$P_f = \frac{96.2\,*V_{40} - 1.5\,P_{3.8}}{90.5}$$

and

$$P_s = \frac{P_{3.8} - 3.8\,*V_{40}}{90.5}$$

An example using this procedure may be shown, provided, again, that the relevant prices and costs are known. If the price of 3.8 milk for Class I purposes is \$5.46 per hundredweight, the jobbing cream price for 40 per cent cream is \$1.20 per pound of fat; and if it is further assumed that the processing and marketing costs of cream—over and above receiving and separating costs—are \$0.02 per pound of fat and that the cost of receiving and separating milk into cream can be arbitrarily allocated so that the cream carries \$0.03 per pound of fat, then the calculations may be carried on as will follow. The first step involves adjustment of the jobbing cream price to a plant entry basis:

$$*V_{40} = (.99)\ (1.20 - .02 - .03)$$
$$*V_{40} = 1.1385$$

Where milk of 3.8 per cent fat is used as a base, the value of fat can be determined:

$$P_f = \frac{96.2 \times 1.1385 - 1.5 \times 5.46}{90.5}$$
$$P_f = \frac{109.524 - 8.190}{90.5}$$
$$P_f = \$1.120$$

while the price of skim milk can be found:

$$P_s = \frac{5.46 - 3.8 \times 1.1385}{90.5}$$
$$P_s = \$0.0125$$

It should be pointed out that these represent "accounting" prices upon which distributors might pay producers for the fat and skim elements contained in whole milk. These prices, for example, are consistent with the established price for whole milk, as can be demonstrated:

$$P_m = 3.8 P_f + 96.2 P_s$$
$$\$5.46 = 3.8 \times 1.120 + 96.2 \times .0125$$
$$\$5.46 = \$4.256 + \$1.203$$

which indicates a slight discrepancy due to rounding off decimals. The butterfat differential consistent with these separate prices for fat and skim milk would be:

$$BFD = 0.1\ (1.120) - 0.1\ (.0125)$$
$$BFD = .1120 - .0012$$
$$BFD = .1108$$

The "direct-ratio-to-fat" butterfat differential, on the other hand, would amount to .1437 for each 1/10 of 1 per cent difference in fat content per hundredweight of milk.

[31] Although this development depends on an assumed base milk price at 3.8 per cent fat test, the method is applicable to any other test.

LOCATION DIFFERENTIALS FOR FLUID MILK MARKETS

The net f.o.b. farm prices received by milk producers supplying a given market normally vary according to the differences in the cost of transporting the milk from farm to market. In this way, near-by producers receive higher net prices than do those located at greater distances from the market. Similarly, the f.o.b. city plant prices between different markets which compete for supplies in a common production area will be expected to differ—by the amount of the difference in transportation costs from the common supply area to the respective markets involved. For this reason, the f.o.b. city plant prices for the different markets in an area such as California will be expected to include a *location differential.* The amount of this location differential depends—as mentioned above—on the amount of transportation cost involved where the *differences* between market prices reflect the *differences* in necessary transportation costs.

In California, the Bureau of Milk Control has established Class I prices for the several markets under control at levels which, in general, reflect these location differentials.[32] The prices at the major deficit consuming areas, such as the Bay Area and Los Angeles, exceed those for markets in outlying areas by an amount which is sufficient to attract an adequate supply to these major markets. To be consistent, the separate prices established for fat and skim milk should also include differentials of this type.

There are logical as well as practical reasons to substantiate the argument for a uniform butterfat differential for fluid milk between various marketing areas which are in close contact with one another. Logically, it follows that, where intermarket transfers are made between areas, the appropriate charges are incurred and should be accounted for on a hundredweight basis rather than by an arbitrary allocation to the separate components—particularly when such shipments typically involve whole milk rather than either cream or skim milk. The practical aspects envisage the inherent administrative difficulties involved in accounting to producers for milk which may have been sold in several separate marketing areas. When such a procedure, involving a uniform butterfat differential, is applied, the separate prices for skim and fat may be determined, in the manner prescribed on page 21, through the following relationships:

$$3.8P_f + 96.2P_s = P_{3.8}$$
$$.1P_f - \quad .1P_s = BFD$$

where P_f and P_s refer to the separate prices for fat and skim milk per pound, respectively; where $P_{3.8}$ is the price per hundredweight for Class I fluid milk of 3.8 per cent fat content, as established for the specific market by the Bureau of Milk Control; and where BFD is the amount of the butterfat differential, as determined by Procedure 2 for the major market—Los Angeles or the Bay Area—which exercises greatest influence on the level of whole milk prices in the area under consideration.

To illustrate the application of the above relationships to determine specific fat and skim prices for an outlying area, assume that the Class I price in *X* marketing area is \$5.35. Assume further that *X* marketing area is within the influence of the Bay Area and that the calculated separate prices for fat and cream in the Bay Area are \$1.12 and \$0.0125 per pound, respectively. These separate prices are equivalent to a butterfat differential

[32] The Bureau of Milk Control currently establishes prices in more than 30 separate marketing areas within the state. These separate marketing areas include virtually every major population concentration, and over 95 per cent of the Grade A milk is sold in areas which are under price control.

which amounts to approximately 11 cents. Upon substitution, we have:

$$3.8P_f + 96.2P_s = \$5.35$$
$$.1P_f - \quad .1P_s = \quad .11$$

or, by solving, $P_f = \$1.11$ and $P_s = .0117$. These then would be the separate prices for fat and skim milk in market X, which are consistent with a price for 3.8 per cent whole milk of $5.35 and with an 11-cent butterfat differential.

SUMMARY AND CONCLUSIONS

This report has centered attention on the problems involved in pricing milk of varying fat content, and on the possibilities of serious inequities—both among producers of milk of high and low fat content and among different distributors. It is the central argument of this statement that the pricing schedule should closely reflect the differential values of the various types of milk. Suggestions have been made as to the manner in which these values may be determined.

With respect to milk entering manufactured dairy products plants, the procedure for establishing the values of milk of different fat and nonfat solids is much more clear-cut than is that for milk sold for fluid uses. In the manufactured dairy products industry, plant returns are directly related to the output of a fairly uniform set of products in a given operation. In turn, product output—or yield—is directly a function of the quantities of solids, both fat and nonfat, present in the milk being processed. Studies made by dairy scientists provide reasonably accurate estimates of the nonfat solids content of milk when the fat content is known. Federal standards specifying the composition of dairy products, together with information on allowances for losses in reasonably efficient plants, make possible the formulation of product yield relationships as a function of the fat test of incoming milk. The net values of these products—prices less processing costs—provide the bases for determining the "worth" or value of this milk to the plant. When applied to the different quantities of products obtained from milk of varying fat and nonfat solids content, these net values provide the information necessary for the determination of the butterfat differential.

For any given plant, an individual schedule could be computed on the basis of the products, costs, and prices actually realized. Unless, however, the prices for the various manufactured dairy products are in perfect relationship—that is, the net values of milk equal in all uses—it will not be possible to determine a single uniform price schedule that will precisely define the values for all types of milk under all types of utilization patterns. In order to provide a single price schedule that can be followed by all plants within a common milk supply area, it may be necessary to make compromises which reduce the accuracy and objectivity of the procedure discussed. Such adjustments and compromises should, of course, attempt to reflect average values based on typical utilization patterns. For example, if substantially different net values result from butter and powder operations than from the production of evaporated milk and other dairy products, the price schedule should be based on values of the products actually manufactured—or possibly on the "highest value" use in order to encourage the industry to use this outlet—rather than on the basis of butter and nonfat solids prices.

The problem of pricing market milk, to determine values in fluid uses, is quite different from that in which manufactured products are concerned. There are three value components of milk used for

fluid purposes: a value for fat; a value for skim milk; and the so-called "Class I premium." The pricing of milk of varying milk fat and skim milk composition largely involves a procedure by which the Class I premium becomes allocated between the fat and skim components. To some extent, at least, the market does provide an opportunity to "price" these elements separately. Others have suggested that the allocation of this premium might be determined by the use of the "alternative value" concept—by accepting the relative values of fat and skim established by the butter and nonfat dried solids markets. Alternatively, the fact that fluid milk distributors may obtain a part of their supplies through different sources—as whole milk, from producers, at prices established by the Bureau of Milk Control, or from other distributors, as 40 per cent cream at the uncontrolled Grade A jobbing cream prices—provides a means for determining how the market is evaluating the *difference* in the physical composition of these two types of supply.

Formulas, as well as examples, are provided to assist in the computation of prices for whole milk entering into manufacturing processes, butterfat differentials under alternative utilization conditions, and separate prices or values for the fat and skim milk elements when the prices for whole milk and those for cream of specified fat content are known. No effort is made to include all of the alternative product combinations, but the principal products made by California dairy plants are covered.

Finally, the possibilities for inequities resulting from the current procedure of accounting for utilization of market milk solely on the basis of fat use should be emphasized. Where milk is paid for on a classified price system, and where Class I products—those which involve payment at the highest price—include both fat-containing and skim milk-containing items, serious differences in the competitive position of distributors may result from this procedure. At present, distributors with high Class I skim milk use, relative to sales of milk fat, receive skim milk, from producers, some of which is used for Class I purposes but paid for at Class II and III prices. Conversely, a distributor with large sales of Class I milk fat relative to Class I use of skim milk is penalized to the extent of having to pay Class I prices for skim milk in excess of his Class I utilization. This problem is particularly acute in the case of country plants that separate milk into cream for shipment to metropolitan markets—in which case the skim milk is retained in the country for manufacturing purposes. Under such circumstances, if distributors are required to account to producers, at Class I prices, for milk used for cream shipments, they would—under current procedures—be forced to pay the prevailing Class I premium on the skim milk associated with cream supplies so obtained. For this reason, it would seem wise for the Bureau of Milk Control to initiate a procedure of establishing separate prices for the fat and skim components of market milk and to require all distributors to account to producers on the basis of the ultimate usage of both of these components of milk.

APPENDIX A

The following tables present calculations which have been based on the formulas and equations presented in the text. For the purpose of these illustrations, it has been necessary to specify certain product prices as well as specific cost allowances. These prices and allowances are given in the footnotes to the tables in which they have been used. It is emphasized that these have been arbitrarily selected and that use of these formulations of costs and prices requires further and special study for purposes of administrative pricing. The prices which have been used all refer to currently appropriate San Francisco quotations. The cost allowances are estimates based on available information on processing costs from published studies, adjusted to reflect current cost levels.[33]

Appendix Table 1 provides a comparison of the net values of whole milk under certain specified uses. From this it can

[33] Processing costs are available for 1943 in U. S. Department of Agriculture, Production and Marketing Administration, Compliance and Investigation Branch, *Milk Products—Costs, Prices and Profits of War Food Purchases,* Washington, D.C., 1946.

Receiving, testing, and separating costs were presented for 1945 by: Thomsen, L. C., "Shall We Plan Toward Continued Diversification?" *American Butter Review,* March, 1945.

These cost allowances have been adjusted to 1953 levels through arbitrary estimates which involve consideration of changes in cost rates of the various cost components such as labor, supplies, building materials, etc.

Appendix Table 1. Comparison of Net Values of Whole Milk in Alternative Uses*

Milk fat	Butter, powder	Evaporated milk	Manufacturing cream and skim milk	Market cream and skim milk
per cent	dollars per hundredweight of milk			
3.0	3.00	3.54	3.53	4.46
3.5	3.41	4.05	3.96	5.04
4.0	3.83	4.54	4.40	5.61
4.5	4.24	4.96	4.84	6.18
5.0	4.66	5.38	5.27	6.76
5.5	5.07	5.80	5.71	7.33
6.0	5.49	6.23	6.14	7.90

* Based on assumed prices and costs as follows:

Prices:

- Butter $0.6850
- Spray process dried nonfat solids 0.1800
- Evaporated milk . . . . . 6.24 per case
- Manufacturing grade cream (40 per cent fat) . . . . . 0.91 per pound fat
- Manufacturing grade skim milk . . . . . 1.19 per hundredweight
- Market grade cream (40 per cent fat) . . . . . 1.21 per pound fat
- Market grade skim milk . . . . . 1.35 per hundredweight

Cost estimates:

- Direct processing and marketing costs:
  - Butter . . . . . $0.05 per pound
  - Nonfat solids . . . . . 0.07 per pound
  - Evaporated milk . . . . . 2.05 per case
- Receiving and separating cost . . . . . 0.20 per hundredweight

NOTE:

Calculated butterfat differentials:

- Butter, powder . . . . . 8.3 cents
- Evaporated milk:
  - Below 3.9 per cent fat . . . . . 10.2 cents
  - Above 3.9 per cent fat . . . . . 8.4 cents
- Manufacturing grade cream and skim milk . . . . . 8.7 cents
- Grade A cream and skim . . . . . 11.5 cents

be seen that, at present price and estimated cost levels, the butter-powder operation results in the lowest net value—or "paying ability"—for all tests of milk. The net values of milk used for evaporated milk and for jobbing sales of manufacturing grade cream and skim milk are at reasonably consistent levels. This is in conformity with the general experience of manufactured milk plants within the state—that these outlets normally provide uniform values for milk of this grade and high values relative to butter and powder. The net values for Grade A cream and skim milk, of course, include the "Class I premium" and so result in higher returns than do the manufacturing uses.

It is interesting to note that these net values increase with increases in the level of fat content of the whole milk received. The amount of the increase in net values provides an indication of the butterfat differential which will reflect the different values of milk of varying fat content. These butterfat differentials are shown below the footnotes in Appendix Table 1, and range from 8.3 cents to 11.5 cents—where the low differential is associated with the butter-powder combination and the high differential includes the Class I premium reflected in the higher prices for milk used for these Class I purposes. The differentials from the manufacturing operations—with the exception of that for milk of less than 3.9 per cent fat content used for evaporated milk—are quite consistent, and range from 8.3 to 8.7 cents.

Appendix Table 2 presents a comparison of the separate Class I prices for milk fat and for skim milk calculated according to the alternative procedures discussed in the text. As can be seen, Procedure 1—based on the relative values of fat and skim milk as determined from the butter and powder market—directly ties the calculated prices for fat in whole milk to changes in butter prices relative to those for nonfat dried solids. Procedure 2, on the other hand, relates changes in these accounting prices for fat in whole milk to the change in the Grade A jobbing cream market. Since these latter cream prices were more stable than butter prices during the period shown, the calculated prices for fat under Procedure 2 show less month-to-month variation than do those estimated with Procedure 1. It is interesting to note further that, when the Class I price for 3.8 milk increased from $5.40 to $5.80 per hundredweight, from January to February, the Grade A jobbing cream price increased only slightly. As a result, the later increase in the fluid milk price became included in the calculated accounting price for skim milk.

APPENDIX B

Formulas to Determine Butterfat Differentials for Fluid Milk, Taking into Consideration the Variations in Nonfat Solids

The principle involved in calculating butterfat differentials for milk for fluid purposes (Class I) which account for variation in nonfat solids is much the same as that presented for similar differentials for manufacturing grade milk used in a butter-powder operation as presented on page 14. This is to say that the butterfat differential is equal to the value of the added fat and nonfat solids in each $\frac{1}{10}$ of 1 per cent difference in fat content per 100 pounds of milk. As determined by the California relation, the physical quantities involved are 0.1 pound of fat and 0.0444 pound of nonfat solids.

The problem of determining the values of these physical quantities when used for fluid purposes—the allocation of the "Class I premium" between components

Appendix Table 2. Comparison of Calculated, Separate Class I Prices for Fat and Skim Milk Based on Alternative Procedures

Month	Butter per pound	Market prices*			Calculated prices			
					Procedure 1†		Procedure 2‡	
		Nonfat solids per pound	Grade A jobbing cream per pound	Class I fluid milk (3.8 per cent) per hundredweight	Fat per pound	Skim milk per hundredweight	Fat per pound	Skim milk per hundredweight
	dollars							
1952:								
January	0.806	0.175	1.20	5.40	1.12	1.19	1.12	1.19
February	0.868	0.175	1.21	5.80	1.22	1.21	1.12	1.59
March	0.808	0.185	1.22	5.80	1.18	1.37	1.13	1.55
April	0.734	0.190	1.22	5.80	1.14	1.53	1.13	1.55
May	0.723	0.190	1.23	5.80	1.12	1.60	1.14	1.51
June	0.724	0.190	1.26	5.80	1.13	1.57	1.18	1.38
July	0.750	0.190	1.26	5.80	1.14	1.53	1.18	1.38
August	0.766	0.190	1.26	5.80	1.15	1.49	1.18	1.38
September	0.775	0.190	1.26	5.80	1.15	1.49	1.18	1.38
October	0.762	0.190	1.24	5.80	1.15	1.49	1.16	1.46
November	0.740	0.190	1.24	5.80	1.14	1.53	1.16	1.46
December	0.708	0.190	1.24	5.80	1.13	1.57	1.16	1.46

* All market prices taken f.o.b. San Francisco for this illustration.

† Applying the relative values of fat and skim milk in a butter-powder operation to the Class I price established by the Bureau of Milk Control, as discussed on page 24 ff. in text. The cost allowances used have been arbitrarily selected at 5 cents per pound of butter and 7 cents per pound of nonfat solids.

‡ Using the Grade A jobbing cream prices to establish fat and skim milk values, as discussed on page 26 ff. in text. It is assumed that processing and marketing costs of jobbing cream (over and above receiving and separating costs) amount to 2 cents per pound of fat—the equivalent of 2.5 pounds of 40 per cent cream. The amount of the costs of receiving and separating whole milk which are allocated to the fat has been arbitrarily established at 3 cents per pound of fat—the equivalent of about 11 cents per hundredweight of whole milk.

as previously discussed—still remains. It is anticipated that this problem may be solved by the use of either Procedure 1 or 2 to determine the separate values of fat and skim milk in fluid milk. In this case, it is necessary to consider that the value of skim milk represents the value of the nonfat solids present in the skim milk for the original level of nonfat solids content of the milk for which these separate values were calculated.

The value of nonfat solids per pound would then be determined by dividing the value of the skim milk by the quantity of nonfat solids present in the skim milk, and the butterfat differential would be:

$$BFD = 0.1P_f + .0444P_{nfs}$$

For example, under either Procedure 1 or 2, assume that the value of skim milk in whole milk of 3.8 per cent fat content had been determined to be $1.38. As the 96.2 pounds of skim milk per hundredweight of whole milk contain 8.757 pounds of nonfat solids, the value of nonfat solids, per pound, would be:

$$P_{nfs} = \frac{1.38}{8.757}$$

$$P_{nfs} = .1576$$

The value of fat (P_f) would be unaffected by this difference in procedure. Assume that this—as determined from previous calculations—is $1.075 per pound. The appropriate butterfat differential would therefore be:

$$BFD = .1\ (1.075) + .0444\ (.1576)$$

$$BFD = .1075 + .0070$$

$$BFD = .1145$$

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For more information: Write to the Department of Agricultural Economics, University of California, Davis.

or: See your University of California Farm Advisor for college entrance requirements.